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The Financial Capital Constraint and the Valuation of Commercial Banking Activity

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Abstract

Equity capital makes the value of liabilities equal the value of assets in a commercial bank's balance sheet. This fact makes valuation of banking activity straightforward. The cost of shareholder risk-bearing becomes a natural entry into the cost side of value-added. Expected losses from lending become an entry in value-added and this leads to the concept of *ex ante* and *ex post* value-added. A model of a profit-maximizing bank is set out and the role of homogeneity in the production functions turns out to be crucial in defining inputs and output. The implicit price of deposit capital is analyzed, including the question of whether deposits are a "cheap" source of funds to the bank.

1. Introduction

A rudimentary conception of a commercial bank is an entity which rents Q_d of capital at rate r_d at one door from depositors and in turn rents out $(1 - k)Q_d$ of this capital at rate r_l as loans at its other door¹. k is the reserve ratio. A simple representation of what banks do is net flow $r_l Q_l - r_d Q_d$, where $Q_l (= (1 - k)Q_d)$ and Q_d are loan and deposit capital respectively. With this flow representation in hand, one can proceed to adjust it in order to measure a bank's value-added in a national accounting sense (as in Fixler and Zieschang [1991]) or to expand the two flows to include inflows and outflows of interest from other assets in the bank's portfolio in order to measure a bank's "efficiency" (as in Rose [1994; p. 111]). Here we note that (a) a commercial bank does not normally exist without equity capital in addition to deposit capital and (b) that when equity capital is introduced into the simple representation of a bank, two central results emerge.² First, the opportunity cost of capital is clarified and one finds that the return to risk-bearing becomes an essential part of value-added (appearing as a type of profit) and secondly, the marginal cost of capital to the bank emerges as a central magnitude, defining in fact, equilibrium bank size. We obtain a clear picture of the value of deposit capital to the bank. We are able to clarify the nature of deposits as being a "cheap" source of capital for the bank³.

¹"What big banks did was acquire money cheaply in the form of deposits and then lend it out at a modest profit to corporations." (Nicholas Lemann in a review entitled "The Man Who Freed the Banks", **The New York Times Book Review**, May 12, 1966, p. 12.)

²In Table 1a below, we observe that the equity to loan ratio varied from .06 to .12 in the years 1981 to 1988 for chartered banks as a group, in Canada.

³Here is an instance of the use of the term "cheap" capital. "A major problem which foreign banks face in host countries in attempts at developing their loan business with local enterprises

We observe in addition that "equilibrium" loan losses become a novel entry in value-added. We are forced to confront the choice between *ex post* and *ex ante* value-added. And finally that the cost of maintaining reserves is seen to be another essential entry on the input side of value-added. This latter entry turns out to be completely analogous to an entry for the cost of inventory maintenance for a non-banking firm and so is less novel than the other two entries, one for shareholder risk-bearing and the other for loss provision. We develop these accounting results first, assuming, as is traditional in this literature, that an equilibrium bank exists. We then turn to the detailed structure of an equilibrium bank with a view to providing a foundation for our accounting arguments. We argue then that equilibrium bank size reflects the target risk level that shareholders will tolerate while keeping their capital in the bank.

It is fairly commonplace to argue that the commercial bank, as conventionally structured, is an institutional form for shifting low cost capital from the pockets of depositors to borrowers. An approach to analyzing this is to consider an alternate institutional form, one in which depositors are removed from the channel of moving capital to commercial and consumer borrowers, as in the "narrow bank" form, which involves deposit funds invested in riskless securities and commercial and consumer lending financed by equity and possibly debt, *sans* deposit capital. Irving Fisher advocated the introduction of this banking form for the United States during the depression of the 1930's.⁴ We consider this institutional form toward the end.

is the fact that access to cheap funding sources, notably retail deposits, is difficult even in the absence of regulatory barriers." (Schijer [1992, p. 24]). See also footnote 1, above.

⁴Miller [1995] makes some comments on the "narrow bank" form.

2. Equity Capital in the Measurement of a Commercial Bank's Activity

In a modern bank's accounts (we have in mind the accounts in the Bank of Canada's publication), our loan income $r_l Q_l$ is labelled **interest income** and Q_l is a vector of loans of different types: commercial, mortgage, consumer, etc. Corresponding to the vector Q_l is a vector of distinct interest rates r_l . On the other side, so to speak, is interest expense. Thus our demand deposits Q_d above is really a vector of deposits of different types: demand, term, saving, etc. And each has its own interest rate, making r_d a vector. Included in interest income are revenues from securities held, particularly government bonds. Hence $r_l Q_l$ includes government bonds. Also included in interest expenses are flows from capital held as debentures or debt issued. So we now have interest income less interest expense in

$$r_l Q_l - r_d Q_d - r_{dd} Q_{dd} \quad (2.1)$$

where r_l , r_d , Q_l , and Q_d are vectors. Q_{dd} are demand deposits and r_{dd} is the demand deposit rate. A basic accounting relation is

$$Q_l - Q_d - (1 - k)Q_{dd} = E. \quad (2.2)$$

where E is equity. This equation is **the financial capital constraint**. First we observe, using equations (2.1) and (2.2) that

$$\begin{aligned} r_l Q_l - r_d Q_d - r_{dd} Q_{dd} - r E &= [r_l - r] Q_l + [r - r_d] Q_d \\ &\quad + [r - r_{dd}] Q_{dd} - r k Q_{dd}. \end{aligned} \quad (2.3)$$

Net interest flow minus $r E$, equals the opportunity cost of capital to depositors, $[r - r_d] Q_d + [r - r_{dd}] Q_{dd}$, plus $[r_l - r] Q_l$, a "surplus" earned on loan capital by

the bank, minus the opportunity cost of capital held as reserves, namely $r_k Q_{dd}$. r is the "certain rate" which we will take as the rate on 90 day government bonds. Equity holders could get r percent on their capital, E , instead of putting their capital in shares of the bank. Similarly, depositors could get r percent instead of r_d . Hence $[r - r_d]Q_d + [r - r_{dd}]Q_{dd}$ is interest foregone to depositors who place their funds in the bank as deposits. We will emphasize below that depositors end up receiving $[r - r_d]Q_d + [r - r_{dd}]Q_{dd}$ worth of services (namely check-writing, safe-keeping, etc.) for this implicit payment⁵. Hancock [1985], following Barnett [1978],[1981], refer to the opportunity costs above as "user costs", a term for asset prices favored by Keynes⁶. For **positive** user costs, Barnett and Hancock refer to the corresponding flows as **outputs**; the label **inputs** is used for flows with **negative** user costs. This approach derives from the practice in the theory of general equilibrium where inputs have negative prices and outputs have positive prices. Here this convention leads to deposits being referred to as outputs from the bank. This makes sense if one recognizes that the bank must provide services, outputs, to providers of capital who receive directly, less than r percent on their capital from the bank⁷.

⁵A different view of a bank is as a type of investment agency. Deposits are viewed narrowly as being placed in the bank to be invested on behalf of their owners. See for example Crouhy and Galai [1991]. There is no acknowledgement of deposits yielding a return in the form of transactions services. Our view is of course very different since we consider that depositors obtain a tangible return in the form of services provided by the bank. These services are a return on top of the return represented by the interest income obtained from the bank for the use of the deposit capital.

⁶Barnett and Hancock introduce an opportunity cost of capital in their user costs for our r , the "certain rate". They focus attention on their opportunity cost and attempt to estimate it. We are treating the opportunity cost of funds as a "certain rate", free of risk and we use the government's 90 day bond rate as a proxy.

⁷Triplett [1992] discusses the perennial question of whether deposits should be viewed as inputs to banking activity or outputs in a useful way. He also briefly surveys approaches to valuing commercial banking services. We recommend turning to Fixler and Zieschang [1991] for

The interpretation of $[r_l - r]Q_l$ is less straightforward, given the user cost approach. rQ_l is of course what the capital Q_l could earn in safe government bonds. r_lQ_l is what it does earn. (We take up losses on loans below.) Hence $[r_l - r]Q_l$ is the **compensation** to the bank for (i)organizing the letting of capital out as loans and (ii) **taking on** the risk associated with risky assets, namely loans. Alternatively, risk-bearing and loan management are **inputs** to the process of loan provision. This provides a partial reconciliation with Hancock's terminology. Loan provision involves $[r_l - r]Q_l$ of inputs, and $[r - r_l]$ is the corresponding negative user cost. We will have more to say of the risk content of $[r_l - r]$ below, and also of $[r - r_d]Q_d + [r - r_{dd}]Q_{dd}$ as a measure of deposit services. We summarize. $[r - r_d]Q_d + [r - r_{dd}]Q_{dd}$ is a payment to the bank by depositors for the bank's provision of deposit services and $[r_l - r]Q_l$ is a payment to the bank by borrowers for the bank's provision of risk-bearing and loan-placement services. Borrowers must pay a risk premium to the lender, the bank. Corresponding to these payments is a cost of production of the services by the bank. These costs of production are taken up below. We turn to the data.

The aggregates, **interest income** and **interest expense**, for all chartered banks in Canada are published. We now consider building up these figures from data on equity, capital, and interest rates. That is, we use the "formula", $(r_l - r)Q_l + (r - r_d)Q_d + [r - r_{dd}]Q_{dd} - rkQ_{dd} + rE$, to re-construct the value, interest income minus interest expenses. We can of course compare the actual value with the constructed value. Our point of departure is that elements of Q_l and Q_d actually earn approximately r percent and these elements can be ignored in our construction since they will cancel in $(r_l - r)$ and $(r - r_d)$. In our construction we

a point of departure in the task of valuing banking services.

need only consider elements of Q_l and Q_d that earn interest at rates significantly different from r percent. First for r , we use the 90 day government bond rate. In Q_l we have a single entry, namely "total loans" earning the prime rate plus 1.5 percent. Demand deposits Q_{dd} earn r_{dd} , a rate equal to interest income for this category, divided by capital value. Q_d comprises "checkable savings desposits" earning $r_{dd} + 1.5\%$; "term deposits" earning $r - 1\%$; and "non-checkable savings deposits" earning $r - 2\%$. Our entries for total loans, term deposits, non-checkable savings deposits, checkable savings deposits, and demand deposits come from CANSIM (the Statistics Canada on-line data base) files. The rates, r , r_{dd} , and the prime rate also come from CANSIM files. The adjustments to these rates are informed guesses⁸. k is chosen as 7%. Equity value E , is from the Bank of Canada data series. Our initial date was determined by data availability. These series were first published then. Our terminal date was determined by the date of revision to the federal law governing banking activity. In 1988 banks entered the brokerage business and such activity would begin to affect traditional activities, namely deposit-taking and loan-placing. We record the data and results in Table 1. (Table 1a contains the data and sources for the figures in Table 1.)

Table 1
Constructed Net Interest Flow

The final column in Table 1 indicates the difference between the published net interest flow for chartered banks in Canada for selected years and our constructed

⁸I have not been able to find the interest rates or interest flows which I require in government sources. Such flows are reported in the annual reports of the chartered banks. One can construct effective interest rates with the interest flow data. My students have done this with a sample of Canadian banks and reported figures similar to my guesses. We are dealing with all chartered banks as an aggregate in our calculations.

value for the flow. Recall that interest rates were very high in 1981 and somewhat less so in 1982 as the Bank of Canada attempted to wring persistent inflation out of the economy. Our poor matches for those years can be explained best by loans being locked in at lower earlier rates. The very high prime rates were not reflective of actual revenue from loans in 1981 and 1982. However our matches for 1983 through 1988 are good. This suggests that our hypothesis about numerous entries in Q_l and Q_d earning approximately r percent was reasonable and in conjunction, that our guesses for relevant interest rates were also reasonable.

A weighted average of the deposit rates, \bar{r}_d , is calculated with the data in Table 1a as weights. This \bar{r}_d is reported in Table 1b. One has then an opportunity cost of deposit funds, $r - \bar{r}_d$, also reported in Table 1b. $r_l - r$ is also reported. Recall that r_l is the prime rate plus 1.5 percent and r is the government's 90 day bond rate. We will comment on these opportunity costs below. In Table 1b we report the difference $r_l - \bar{r}_d$ also. We note here that the opportunity cost of deposit funds is relatively low. We leave the data for a while.

rE is in fact less than what a bank pays for equity capital. A valid measure of the cost of equity is net profit (net of taxes, depreciation and loan losses) plus dividends. We refer to this as y . This y includes a compensation to the shareholders for risk-bearing. Shareholder asset equilibrium satisfies the arbitrage condition

$$y + \Delta E = (r + \gamma)E \tag{2.4}$$

where γ is the risk premium expected by equity holders. $\Delta E = E_{t+1} - E_t$. Hence shareholder value E_{t+1} will adjust to current income y . One can view ΔE , roughly speaking, as realized risk to the shareholder. γ can be interpreted as the risk rate corresponding to what shareholders expect to bear. γ will change in response

to persistent unanticipated values of ΔE . For a hypothetical steady state bank, y will be constant, on average and ΔE will be small and equal to zero, when averaged over time. In the steady state bank, risk will reflect volatility in E , or **liquidity risk**, not the risk of a capital loss which one associates with the bankruptcy of the bank. The shareholder's "choice" of γ is based on leverage and the volatility of the bank's loan portfolio. There is no reason to expect that γ will be the same across banks. y/E is a widely-used measure of a bank's performance and since it equals $(r + \gamma) - \Delta E/E$, we do not expect this measure to be equal across banking firms. Booth [1996] refers to $(y/E) - r$ as a measure of shareholder risk. It is really a net measure, since it equals $\gamma - (\Delta E/E)$.

It follows from our relations above ((2.1),(2.2), and (2.4)) that

$$\begin{aligned}
r_l Q_l - r_d Q_d - r_{dd} Q_{dd} - y &= r_l Q_l - r_d Q_d - r_{dd} Q_{dd} - (r + \gamma)E + \Delta E \\
&= [r_l - r]Q_l + [r - r_d]Q_d + [r - r_{dd}]Q_{dd} \\
&\quad - r k Q_{dd} - \gamma E + \Delta E.
\end{aligned} \tag{2.5}$$

That is, interest revenue minus interest expense and the payment for equity equals the premium payment on loans, $[r_l - r]Q_l$ by borrowers, plus the payment for deposit services, $[r - r_d]Q_d + [r - r_{dd}]Q_{dd}$ (or opportunity cost of deposit capital) minus the net cost of risk bearing by shareholders minus the opportunity cost of reserves. γ is the risk premium demanded by shareholders and ΔE are current capital gains. The presence of a non-zero ΔE reflects the fact that the bank is not in a steady state. We will label $\gamma E - \Delta E$ as the bank's payment to equity holders for risk-bearing and place it on the input side of the bank's accounts, when we take up this matter below. Since $[r_l - r]Q_l$ is a payment to the bank for taking on loans and, in equilibrium, equals the bank's cost, we have $[r_l - r]Q_l - [\gamma E - \Delta E]$

as the bank's cost, net of shareholder risk-bearing. Since "the bank" cannot bear risk, only its stakeholders can, $[r_l - r]Q_l - [\gamma E - \Delta E]$ includes a payment for residual risk borne by borrowers in the form of a mark-up in the rates charged for loans and by depositors who, given deposit insurance, pass the risk on to the insurers. There is also a component here for compensation for the expenses of loan-placing *per se*. We focus on these matters below when we analyze a model of a profit-maximizing bank.

Equation (2.5) is our basic decomposition of a bank's net interest flow. By completing the measure so as to include the cost of equity we obtain: net capital flows equal the payment to the bank for risk-bearing and loan-placing by borrowers, the payment to the bank for deposit services by depositors minus the payment to equity holders for risk-bearing. The bank sells risk-bearing and management services to borrowers, at a price, and passes some of the risk-bearing back to shareholders, the owners of the bank. Other risk-bearing costs are absorbed by borrowers, or more precisely, the consumers of products of the borrowers' businesses. And some risk is passed back to depositors, or their insurers. This is made clear below. Our decomposition links the opportunity cost of capital to the value of outputs of the bank in a useful way. The decomposition is based on a completion of the capital flow analysis of a bank – the inclusion of shareholder equity. With minor adjustments, taken up below, $[r_l - r]Q_l$ and $[r - r_d]Q_d + [r - r_{dd}]Q_{dd}$ provide good measures of a bank's production of services. Moreover, given some averaging, one can express vectors r_l and Q_l by scalars, and similarly for r_d and Q_d , and obtain good approximations of the value of bank production (a bank's value-added). At this point we have the input measure of value-added as: labor plus structure and machine capital costs, plus the cost of risk-bearing by equity

holders plus the costs or reserve holding. The output measure of value-added is $[r_l - r]Q_l$ plus $[r - r_d]Q_d + [r - r_{dd}]Q_{dd}$. These latter constitute payments by borrowers and depositors respectively for services from the bank. We expand on this below.

We have, then, by introducing equity capital into the basic "model" of a bank, obtained two significant results. We have the return to risk-bearing as an equilibrium entry for profit on the input side of value-added and we have obtained a natural decomposition of value-added on the product side into the value of services to depositors and the value of services to borrowers.

3. Refinements to the Measure of Value-added

There are obvious refinements needed to $[r_l - r]Q_l + [r - r_d]Q_d + [r - r_{dd}]Q_{dd}$ as a measure of a commercial bank's value-added. We need entries for service charge revenue, associated principally with deposit administration, and for loan losses. Also we require an accounting for deposit insurance premiums paid by the bank. We proceed to treat these matters from an accounting perspective. It is convenient to aggregate loans into a single entity and to add government securities, B_G to the assets. In addition, it is convenient to aggregate deposits into a scalar for demand deposits, namely Q_d and to introduce a separate scalar for savings and term deposits combined, Q_T . Loans Q_l earn r_l and demand deposits earn r_d . We assume however that government securities earn the certain rate r . And we assume that the term and savings deposits also earn r percent. These simplifications (aggregations) imply that equity capital must be

$$E = Q_l - (1 - k)Q_d + B_G - Q_T$$

where k is the equilibrium rate of reserve retention. Reserves are of course held to service depositors who wish to withdraw cash. The assumption that B_G and Q_T each earns r percent leads to these terms cancelling out below and yields our simple expression for value-added on the output side.

If r_l is the nominal charge for loans, we can represent the return, net of loan losses, by $(1 - \alpha)r_l$ where α is a measure of current loan losses, accounting for both interest not paid and capital currently "written down". Finally, we introduce additional net revenue $(s - i)Q_d$ on demand deposits, where s is the rate of service charges and i is the rate of deposit insurance. $i[Q_T + Q_d]$ is the aggregate insurance costs per period, say per year. With these refinements, value-added above, becomes

$$[(1 - \alpha)r_l - r]Q_l + [r - r_d + s]Q_d \quad (3.1)$$

on the output side, and the profit or shareholder risk premium term becomes $\gamma.[Q_l - (1 - k)Q_d + B_G - Q_T]$. This latter is an entry on the input side of value-added. We have omitted capital gains from consideration to simplify matters. We assume that we are dealing with a hypothetical steady state bank; that is, $y = (r + \gamma)E$. rkQ_d is the flow cost of maintaining reserves to service depositors. It goes on the input side of value-added and is a cost completely analogous to the cost of inventory maintenance for a non-banking firm. Insurance costs, $i[Q_T + Q_d]$, are a cost of production and enter the input side of value-added. The core of the input side of the accounts is the value of primary inputs, namely $w_l N_l + w_d N_d + qK$, where N_l is labor employed in loan evaluation, placement, and monitoring, N_d is labor employed in deposit-taking and processing, w_l is the wage of workers on the loan side, w_d is the wage of workers on the deposit side, K is machine and building capital (K -capital), and q is the rental price of K -capital.

Average loan losses, $\alpha r_l Q_l$, raise subtle accounting issues. When they appear on the product side as above, we label the resulting value-added, *ex post* value-added, *ex post* because average loan losses have been netted out; that is, the loan losses have been realized. $[r_l - r]Q_l + [r - r_d + s]Q_d$ is then *ex ante* value-added on the product side. One recognizes that non-transitory cost $\alpha r_l Q_l$ exists in producing loans. The net benefits of loans correspond to the *ex post* concept. However, borrowers see *ex ante* benefits deriving from their proposed projects when they receive their loans. After the fact, some funded projects succeed and a predictable fraction of funds lent end up in failed projects. The *ex post* concept seems to capture the traditional concept of social value—outputs count. But this raises the problem of resources expended (costs) not appearing on the input side of value-added. Net national product (NNP) will be smaller under the *ex post* concept. It is as if the product shrank or evaporated somewhat between the beginning and the end of the accounting period.

With the *ex ante* concept, loan losses become a cost of production and appear on the input side of value-added. Measured NNP will be larger with this approach. In a sense, the input side of value-added is better defined with the *ex ante* concept, while the output side suffers from a type of value inflation. With the *ex post* approach, the product side is better defined but the input side suffers from a type of incompleteness.⁹ We summarize now by reporting value-added in two tables,

⁹This matter is distinct from the controversy in the measurement of the output of the insurance industry (eg. Diewert [1994; p. 10]). There, the issue is whether premiums net of payouts should be in the measurement of output or premiums gross of payouts. For the insurance case, there is not the matter of dealing with **planned** or "permanent" losses. Dan Usher suggests to me that the *ex post* concept is the valid and practiced approach since every firm budgets for losses, as in remaindering some of its current product. This may be true although I am unaware of sources on this topic. Our view is that financial services firms are different because incurring losses is part of doing their "producing". For widget producers, all output is expected to be sold

one for the concept *ex post* and one for the *ex ante* concept.¹⁰

and to cover costs of production, in our view. There is no anticipated residue of unsold product as we argue is inherent in bank lending.

¹⁰Fixler and Zieschang [1991] endorse our *ex ante* output measure for value-added. They do not address the question of equilibrium loan losses in the accounts. On page 63, note 12, they argue that $Q_t - (1 - k)Q_d$ is approximately zero in reality and thus our *Er* term can be ignored. It is our view that in confronting the fact that equity finance is always present, many subtleties in valuation emerge and must be dealt with.

Table 2
Ex Post Value-added
(Chartered Banks in Canada 1984*)
(in millions of dollars)

INPUTS	OUTPUTS
$w_l N_l + w_d N_d + qK$	$[(1 - \alpha)r_l - r]Q_l$
$+ [Q_d + Q_T]i$	$+ [r - r_d + s]Q_d$
$+ krQ_d = 7425.3$	$= 9997.7$
$\gamma[Q_l + B_G - [1 - k]Q_d - Q_T]$	
$+ \text{taxes } (701.7) = 2572.4$	
sum = 9997.7	

Table 3
Ex Ante Value-added

INPUTS	OUTPUTS
$w_l N_l + w_d N_d + qK$	$[r_l - r]Q_l$
$+ [Q_d + Q_T]i$	$+ [r - r_d + s]Q_d$
$+ krQ_d = 7425.3$	$= 12040.8$
$\gamma[Q_l + B_G - [1 - k]Q_d - Q_T]$	
$+ \alpha r_l Q_l (2043.1)$	
$+ \text{taxes } (701.7) = 4615.5$	
sum = 12040.8	

(*Data Source: Bank of Canada Review, February 1986, Chart K2, p. S151)

Tables 2 and 3 indicate that value-added is significantly affected by the placement of losses. These are realized losses, of course, not average losses over multiple periods. We note also that value-added can be as high as 12/444 as a fraction of gross domestic product. If one views banks as providing transacting services (placing of savings in investment projects (intermediation) and providing check-

writing services to depositors), this suggests that such transactions are not an insignificant part of aggregate economic activity.¹¹

There is another way of thinking about average loan losses. The predictable volume of losses resembles economic depreciation of the bank's stock of financial capital. A gross volume is lent out, some is lost to metaphorical "wear and tear", and a net volume is left at the end of the period. In this view *ex post* value-added resembles net national product and *ex ante* value-added resembles gross national product. It is of some interest that J. R. Hicks used the terms *ex ante* and *ex post* in his discussion of net and gross measures of product when he was expositing his well-known concept of Hicksian Income. Recall that NNP is a type of Hicksian Income.

4. The Profit-maximizing Commercial Bank

There is, of course, a model of a commercial bank underlying our accounting entries above. One has to link inputs to outputs via a technology, a production function. In our case, two production functions—one for deposit services (check-writing by depositors and safe-keeping of depositors' funds), $Q_d(N_d, K)$, and one for loan-placing, $Q_l(N_l, K)$. Deposits and loans are each "aggregated" to scalar values. The bank maximizes profit

¹¹There is another way to think about the value of the banking sector. When the banking sector is not functioning smoothly (eg. there are persistent failures of banks on a large scale), there are substantial costs represented by current outputs being less than potential outputs. One could think of the marginal damage caused by x dollars of bank failures. The social costs of x dollars of bankruptcy in the banking sector appear to be significantly higher than the same x dollars in the goods production sector. This is a reflection of the system-wide damage to confidence caused by a bank failure. The decline in confidence can be interpreted as external to the bank and being external, opens the door to some form of central "co-ordination" as in public regulation.

$$\begin{aligned}
\pi = & (1 - \alpha(N_l))r_l Q_l(N_l, K) - (r_d + s - i)Q_d(N_d, K) \\
& - (r + \gamma)\{Q_l(N_l, K) - [1 - k]Q_d(N_d, K)\} \\
& - w_l N_l - w_d N_d - qK
\end{aligned} \tag{4.1}$$

by choice of N_l , N_d , and K . This operation yields three conditions for the efficient use of non-financial inputs and the solution for values of N_l , N_d , and K allow us to obtain equilibrium levels of deposits, loans, and equity finance. In addition, we postulate perfect competition, small firms relative to the market, and zero excess profit for each firm.¹² Finally, to link our equilibrium bank to our accounting above, we require a mechanism for product exhaustion or having the value of inputs equal the value of outputs. The most satisfactory approach is to have the value of outputs equal the value of inputs under an assumption of homogeneity in the production functions or the exhaustion of product under marginal cost pricing of inputs. We will assume this to hold and discuss it further below.¹³ The role of savings deposits and holdings of government bonds will be returned to then. We can say now that we have, in brief, linked the optimizing and efficient bank to our accounting magnitudes above. Details of optimal scale and equilibrium shareholder risk-bearing will be taken up below.

The interesting result to emerge in this equilibrium is the marginal cost of cap-

¹²Many observers note that relative to its local market area, a bank is typically large and has some price-setting power. See for example Berger and Hannan [1989] and Hannan [1991]. We make some comments on this below but our model ignores the possibility of excess profits being obtained because of local market power.

¹³The other approach is to select the degree of homogeneity in advance and to deal with a gap between the value of inputs and outputs under marginal cost pricing of inputs. We wish to avoid the issue of arriving at a compelling gap-filling scenario here although this is a large topic and one at the heart of accounting and the analysis of income distribution.

ital for lending. The bank as lender is indifferent between raising an additional dollar of capital for lending from equity-holders or from depositors. This suggests that though deposit capital seems to be the low cost alternative, it is employed until its marginal cost to the loan side of the bank is the same as the marginal cost of equity capital, this latter being traditionally viewed as being high cost. This equal implicit price for loan capital from the two alternate sources might imply that each type of capital was bearing the same risk in some sense. This is not the case and this observation is fundamental to our analysis of the traditional bank form. Though borrowers implicitly compensate equity holders and depositors equally for risk-bearing, the bank passes on this compensation only to shareholders. The risk compensation for depositors is paid by borrowers but gets passed through to deposit **acquisition** at the margin. It does not show up as a risk premium received by depositors. This will become clear from Figures 1a and 1b below.

We will assume that the bank can affect its loss rate α by varying the number of people it hires on the loan side of the bank. The larger is N_l , relative to Q_l , the more personnel are available to scrutinize loan applications and to monitor the activities of borrowers, downstream. A simple formal statement of this is having $\alpha(N_l)$ a declining function of N_l . Moral hazard in borrowing involves the borrower failing to use the proceeds of the loan prudently. Increased monitoring by the bank mitigates its exposure to the moral hazard problem.

We turn now to the efficiency conditions (first order condition for profit maximization). First we should be precise about units. Q_l and Q_d should be viewed as Q_l and Q_d pieces of paper with one dollar written on them. That is they are physical quantities. Each then is multiplied by a price, namely \$1. Though a bank

is not a printing press for currency, it is useful to think of it producing a quantity of stuff with a dollar value as price multiplied by a physical quantity. The production processes are for producing numbers of loans and quantities of services associated with volumes of deposits. The financial capital involved is processed by labor and capital to become loans in one side of the bank and on the deposit side, transaction and safe-keeping services are generated with labor and capital inputs (as in people, computers, pens, paper, etc.) and the volume of services is tied to the depositor's volume of deposits. It is not accurate to say that deposit capital is an input to the bank's provision of deposit services. Rather, a depositor's volume of deposits represents an indirect payment for a volume of services, and the volume of services is constrained by the level of implicit payment. Reserves, however, are a part of deposit capital that constitute an input into the production of deposit services. Reserves constitute an inventory to be drawn on in order to provide deposit services.

The equilibrium conditions are

$$(1 - \alpha)r_l - (r + \gamma) = [w_l + r_l Q_l \frac{d\alpha}{dN_l}] / (\partial Q_l / \partial N_l) \quad (4.2)$$

$$[1 - k](r + \gamma) - r_d + s - i = w_d / (\partial Q_d / \partial N_d). \quad (4.3)$$

and

$$\begin{aligned} & [(1 - \alpha)r_l - r][\partial Q_l / \partial K] + [(1 - k)r - r_d + s - i][\partial Q_l / \partial K] \\ & - \gamma \{ \partial Q_l / \partial K - [1 - k][\partial Q_d / \partial K] \} = q \end{aligned} \quad (4.4)$$

These conditions allow us to sketch the deposit side of the bank in a price/quantity diagram, namely Figure 1a.

In Figure 1a, $r_d Q_d$ is the rental for deposit capital paid by the bank to depositors. $[(1 - k)(r + \gamma) + s - i]Q_d - r_d Q_d$ represent costs to the bank of servicing the deposits of the depositors. These comprise labor costs (variable costs) and K-capital costs. We ignore land or site costs. Since only fraction $[1 - k]$ of each dollar of deposits is lent to the loan side of the bank for lending out, $[1 - k](r + \gamma)Q_d$ is revenue to the deposit side transferred from the lending side of the bank. $(s - i)Q_d$ is revenue from service charges assessed on deposits, net of deposit insurance costs to the bank.

A sketch of the loan side, based on the equilibrium conditions, is in Figure 1b. $(r + \gamma)Q_l$ is the rental for capital for lending in Figure 1b. Capital $[1 - k]Q_d$ is obtained from the deposit side of the bank and the remainder is obtained from shareholders as equity. $(r + \gamma)$ can be viewed as **the transfer price** which the bank assigns to capital being rented from the deposit side or as the shadow price (rental) on deposit capital. This price of capital for making loans does indeed determine the volume of loans lent out. Marginal cost, inclusive of loan-placing costs, equals net marginal revenue $(1 - \alpha)r_l$. The cross-hatched area in Figure 1b is the cost of risk-bearing by shareholders and goes on the input side of value-added. The top rectangle is the average losses, $\alpha r_l Q_l$. These losses are passed on to borrowers in a "mark-up" in the charge r_l . The term risk-sharing between the bank and the borrowers is appropriate. The novel aspect of risk in Figure 1b is that borrowers compensate depositors for risk-bearing and depositors fail to receive the compensation in the form of a risk premium. $[(r + \gamma) - r] \times (1 - k)Q_d$ in Figure 1b is the payment by borrowers to depositors for risk-bearing. But this revenue is not transferred to the deposit side as a risk premium. It is transferred as

revenue to pay for more inputs into deposit acquisition and processing; that is as an expansion of deposit-taking, and not as a compensation for risk-bearing. This can be seen from an examination of the situation depicted in Figure 1a. There, there is no "surplus" representing compensation for risk-bearing—no surplus of the sort in Figure 1b (the cross-hatched area), going to shareholders as compensation for risk-bearing. If $\gamma[1 - k]Q_d$ were paid to depositors as a compensation for risk-bearing, there would be fewer deposits taken into the bank, at the margin. We provide a sketch in Figure 2 of a hypothetical situation in which depositors are compensated for risk-bearing.

This is an artificial "equilibrium" and is only introduced to drive home the point that with the integrated form, depositors do not end up receiving the return to risk-bearing that is paid by borrowers in the loan side of the bank. The cross-hatched area is a hypothetical payment to depositors for risk-bearing. We have made it a premium over the former market rate, r_d . The integrated bank is an entity that does allow for the "exploitation" of "cheap" deposit capital by having such a bank use more deposit capital at the margin than it would if depositors were paid the risk-premium which the borrowers have already paid in on the loan side of the bank. Depositors end up being protected with deposit insurance, of course, and we have them paying for the insurance in the form of a lower implicit price being paid for their deposit capital by the loan side of the bank.

The model of a competitive profit-maximizing bank, above, provides a foundation for our accounting procedures in the previous section. There remain three loose threads: product exhaustion at marginal cost pricing, the role of term and savings deposits, and government bond holdings in the equilibrium, and equilibrium scale. These last two matters are related and we deal with them in the next section. With regard to product exhaustion (value of inputs equal to the value of outputs), matters will not work out in a simple fashion. We would like to have area $\{[(1-k)(r+\gamma)+s-i]-r_d\} \times Q_d$ in Figure 1a and area $[(1-\alpha)r_l-(r_l+\gamma)] \times Q_l$ in Figure 1b exactly reflect the value of inputs, namely $w_l N_l + w_d N_d + qK$. As it stands these areas represent the value of outputs. We take up the simpler case first. We assume temporarily that the loss rate α is independent of N_l . We then introduce the following **assumptions** on the production functions, that is we **assume** homogeneity in the technology in the following sense:

$$(\partial Q_l / \partial N_l) N_l + (\partial Q_l / \partial K) K \text{ is equal to } Q_l$$

and

$(\partial Q_d / \partial N_d)N_d + (\partial Q_d / \partial K)K$ is equal to Q_d .

These assumptions, combined with our first order conditions, imply that product will be exhausted at marginal cost prices. That is

$$[(1 - \alpha)r_l - (r_l + \gamma)]Q_l + [(1 - k)(r + \gamma) + s - i - r_d]Q_d$$

equals the payments to labor and capital, namely, $w_l N_l + w_d N_d + qK$. Needless to say, our assumptions on the production functions are stringent. We are building in a version of constant returns to scale. Most observers favor the presumption of increasing returns to scale in banking. A larger bank can exploit diversification more effectively, for example. Our sharing of K-capital by the two sides of a bank is a form of increasing returns (the term economies of scope does not seem inappropriate since one stock, namely K, is being used to produce two outputs). It is a quite general result that our homogeneity assumptions on technology (a form of Euler's theorem) break down under increasing returns to scale, scale effects being embodied in the production functions. The value of inputs will exceed the value of outputs under increasing returns and marginal cost pricing of inputs. Fixler and Zieschang for example do not address the issue of product exhaustion and non-constant returns to scale. There is in fact an established line of research which has measured returns to scale in banking using translog cost function specifications. Some scale economies are reported (eg. Hancock [1985], Berger and Humphrey [1992]).

Let us circle back to our specific model and re-introduce the dependence of the loan rate α on labor in the loan side, N_l . Our homogeneity assumptions above will not yield product exhaustion under marginal cost pricing. An extra

worker now makes a marginal contribution to output via $\partial Q_l / \partial N_l$ and via loan loss reduction, $-d\alpha/dN_l$. Our homogeneity assumptions yield the result that the value of inputs exceed the value of outputs by $-N_l r_l Q_l (d\alpha/dN_l)$, the value of labor in loss reduction. At this point one is confronted with the choice of abandoning homogeneity in the technology or the assumption of marginal cost pricing of inputs. Clearly, adjusting the technology to get product exhaustion under marginal cost pricing is the simplest approach since it allows us to hold onto to our analysis centered on Figures 1a and 1b. If we stick with homogeneity, as specified above, we must find an output category to be drawn upon in order to pay the extra return to labor. This is analogous to the problem of obtaining product exhaustion under increasing returns to scale in more familiar models of firm structure. We will assume at this point that the homogeneity of the technology adjusts itself as if by magic in order to preserve product exhaustion under marginal cost pricing of inputs. We are sweeping complicated problems in input valuation under the carpet. We have set the issues out quite explicitly and have taken an easy route around the intrinsic difficulties¹⁴. The difficult route is to construct a theory of input pricing different from marginal cost pricing, under assumptions of non-constant returns to scale.

Note that our assumption of joint use of K-capital by the two sides of the bank is itself somewhat novel. But the problems of product exhaustion under the marginal cost pricing of inputs are not artifacts of our formulation of technology.

¹⁴Since $\alpha(N_l)$ is decreasing in N_l , we could allow for some increasing returns in either loan-placing or deposit processing and still obtain product exhaustion under marginal cost pricing of inputs. With the assumption of zero profits, one could in turn obtain an optimal scale for the bank as the $d\alpha/dN_l$ effect is traded off against the scale effect. Alternatively, one could simply stick with homogeneity and assume that the $d\alpha/dN_l$ effect is small. Scale will then be determined by the value of γ , which we discuss below.

The problems turn on generic issues in specifying the technology (production functions).

5. Bank Size and Risk-bearing

Our bank is really scale free to this point. Here we argue that shareholders adjust the risk premium which they demand in order to be comfortable with the portfolio of the bank. The bank takes γ as parametric when in fact it changes with the structure of the assets of the bank¹⁵. This leads to a simultaneous fitting of the portfolio by the bank to the premium γ which the bank must meet. The bank's ultimate target is an acceptable leverage value and so as the target γ is reconfigured, so also is the ratio of equity to other sources of bank finance.

Shareholders focus on distinct risks – irredemable capital loss and simple volatility in the value of their capital. We have assumed that our steady state bank will not go bankrupt and it covers average losses out of interest charges on loans. Hence shareholders face only risk from volatility in the value of their capital. This we referred to as liquidity risk. The higher the leverage, the more risk will be borne by a share and the more expensive capital for lending will be to the bank. Recall that $r + \gamma$ is the marginal cost (rental price) of loan capital and γ is the risk premium which a shareholder demands. Thus banks prefer *ceteris paribus* a high volume of equity capital in order to lower leverage. And the banks can lower risk to shareholders by funding on average safer projects. Thus the banks can adjust their loan portfolios to meet a target value of leverage. This we discuss further below when we consider a bank drawing on capital from savings

¹⁵If γ is increasing in E our analysis does not change. The bank's prices for intermediation and deposit processing will include a new term, reflecting the rising cost of equity.

and term deposits and lending to the government.

A powerful risk mitigation instrument is diversification across distinct markets, particularly geographically distinct markets. Branch banking is of course a way to effect diversification across regional submarkets and one might argue that the long-run durability of large Canadian banks is a consequence of their branch structure. To the extent that local branches have some limited price-setting power and some local monopoly profit, a branch system can mitigate system-wide risk by pooling local profits and maintain a central fund to bail out local branches with transient liquidity problems. This is somewhat different from the classic diversification associated with uncorrelated ups and downs in activity in distinct submarkets.

Aggregate shareholder risk is affected by the volumes of term and savings deposits taken on by commercial banks. In fact term and non-checkable savings deposits comprise about 70 percent of total deposits of commercial banks in Canada and thus fund a large fraction of loans (see Table 1a for the data). Recall that when banks lend B_G to government at r percent and borrow from the private sector, Q_T , as term and savings deposits at r percent, the net income is $[B_G - Q_T]r$. The volume of capital raised as equity is now $Q_l + B_G - [1 - k]Q_d - Q_T$ with cost to the bank of $[r + \gamma]\{Q_l + B_G - [1 - k]Q_d - Q_T\}$. The net cost of equity becomes

$$\gamma\{Q_l + B_G - [1 - k]Q_d - Q_T\}.$$

(We assume that no reserves are required for the term and savings deposits, though deposit insurance must be paid on these items.) The direct impact of term and savings deposit finance is to allow the bank to put out a larger volume of loans. This capital also provides the bank with room to manoeuvre in selecting a

target level of equity in the bank. A rough characterization of decision-making at the highest level in the bank is that after loading up with deposit capital, both demand and term and savings, lending and bond-holding are adjusted so that a target volume of equity is invested in the bank. The risk premium γ is treated as parametric by the bank, though shareholders revise their demand price (i.e. $r + \gamma$) as their exposure to risk changes.

6. $r_l - r_d$ as a Measure of a Bank's "Efficiency"

There is a tradition of viewing $r_l - r_d$ as measuring the efficiency with which a bank uses deposit capital to fund lending activity.¹⁶ If $r_l - r_d$ is small, it reflects low mark-up over the cost of a dollar of deposits in getting the dollar out in a loan. However, the bank is more than a way-station for deposit capital, it is also a supplier of transactions and safe-keeping services. Thus it is wrong to identify $r_l - r_d$ with the marginal cost of taking a dollar in as a deposit and lending it out as part of a loan. $r_l - r$ is the gross marginal cost of placing a dollar of capital out in loans. $(1 - k)(r + \gamma) + s - i - r_d$ is the marginal cost of providing deposit services on a dollar of deposits. This is net of rental r_d paid to depositors. The sum of these marginal costs is

$$r_l - r_d + \gamma - (r + \gamma)k + (s - i).$$

Note that $r_l - r_d$ may or may not be a good approximation to this value. $r_l - r_d$ could be relatively small, as for Japanese banks, and the marginal cost of

¹⁶"First they record all their bank's interest income from loans and security investments. Then the total interest paid out on borrowed funds is subtracted to derive each bank's net interest income or interest margin. The interest margin measures how efficiently a bank is performing its function of borrowing and lending funds." (Rose [1994, p. 111]).

moving a dollar of deposits into the bank and out as a successful loan could still be relatively large. This would occur if the risk premium γ was large, for example. Note that a high service charge rate s , here "translates" into more deposits being taken in at the margin, and a corresponding increase in marginal costs of deposit-processing; not into excess profits. In Canada, service charges as a component of commercial bank revenue have been growing fairly rapidly since the early 1980s.¹⁷

A candidate for a measure of efficiency of a bank in loan-placing is $[w_l N_l / [(1 - \alpha)r_l Q_l]]$, the fraction of loan-placing labor costs in loan revenue for a dollar lent to a successful project. This indicator will generally increase with the loss rate α . Hence a low value of the indicator will reflect an efficient lending process. The efficiency of taking on a dollar of deposits is a completely different notion. There is of course some small cost of simply processing a new dollar of deposits. Significant costs occur in the process of providing check-writing services. The volume of services per dollar presumably increases with the implicit price $r - r_d$. Thus a relatively large $r - r_d$ could result from high costs in the provision of services or from a large volume of efficiently provided services. One needs some measure of the value of output relative to the value of inputs in order to measure properly the efficiency of deposit-taking and processing. These considerations, above, imply that $r_l - r_d$ or refinements to it is a poor indicator of "the efficiency" of a bank's activity. $r_l - r_d$ reflects two quite different lines of banking activity.

¹⁷Service charges for Canadian chartered banks rose from 0.06 in 1984 to 0.10 in 1988 as a fraction of bank operating revenues (Statistics Canada data assembled by my student Teresa Lee). In 1988 large banks had entered the brokerage business. This ratio rose to 0.14 in 1995.

7. Marginal Payment by the Bank for Risk

It is one thing to say that a shareholder demands a risk premium for lending funds to the bank as equity, and another to say that the bank has to re-coup the cost of risk in its charge for a loan. Both are true statements but they are related somewhat indirectly. In our model, the mark-up $r_l - r$ does in part reflect the risk associated with a dollar lent out but it comprises three parts, the risk borne by the shareholder, which we are referring to as liquidity risk, the risk borne by the borrower in the mark-up in the loan rate representing the expected loss on a dollar lent out, and the management costs of loan placing and monitoring. αr_l of loan revenue is on average lost because of loan risk and γ is the premium paid to a shareholder for a dollar of equity. These comprise the two risk components in $r_l - r$. The third component is the management cost, $(1 - \alpha)r_l - (r + \gamma)$.

These observations might lead one to infer that the two parts of the cost of risk-bearing should be treated the same in the national accounts. This view implies that our *ex ante* concept of value-added is the correct one. We do not find this tilt in favor of the *ex ante* concept to be sufficiently compelling to result in the *ex post* concept in being driven from the field.

Our model implies that γ must lie between r_l and r . In Table 1b we observe that this gap ranges between 2.46 and 3.5 percent. These seem like reasonable values for a risk premium for a bank well back from the edge of bankruptcy. However, these are the outer limits for γ . There are other expenses which our model indicates that the bank is paying out of the "mark-up" $r_l - r$, namely loss provision and lending administration. This suggests that the gap $r_l - r$ is excessively tight to cover off the shareholders' risk premium. Booth [1996] argues

for a 3.5 to 4 percent value for a reasonable risk premium. He observes that profit over equity yields such a premium over the certain rate. In light of our data, it appears that these premia are being financed out of sources distinct from loan-placing *per se*. Profit from the deposit side are a possible source of funds to cover off payment for the risk premia. In this regard, note that though the $r - \bar{r}_d$ values appear relatively small in Table 1b, the associated implicit revenue stream, $(r - \bar{r}_d)\bar{Q}_d$, has a large base, namely \bar{Q}_d , total deposits. Hence this revenue and/or revenue from service charges, may be adequate to assist in financing the costs of risk-bearing by shareholders. However, such cross-subsidization is not a property of our model of the optimizing bank.

A bank facing some risk of bankruptcy will be obliged to pay a higher risk premium on equity. We do not pursue this case since, roughly speaking, it involves non-steady state analysis¹⁸. In the steady state view, all loan losses are shifted to borrowers in the price of a loan. There is a clean separation of risk-bearing between borrowers, shareholders and deposit insurers. The possibility of bankruptcy shifts part of the responsibility of shouldering losses to shareholders and deposit insurers and one has a complicated division of risk-bearing between borrowers, shareholders, and deposit insurers. By definition, under the possibility of bankruptcy, provisions for loan losses fall short of current losses by more than the value of shareholder equity. Shareholders walk away under limited liability with their capital valued at zero and depositors are stuck for the difference between current debt and the market value of the loans.¹⁹ Depositors then turn to

¹⁸The familiar rendering of portfolio instability of a bank is that it is funding assets with longer term maturities with capital (deposits) of short term maturity.

¹⁹In Hartwick[1996], there is a formal analysis of how the limited liability of shareholders in covering losses is capitalized in the value of the equity. Giammarino, Schwartz, and Zechner [1989] provide estimates of how the prospective bankruptcy of the Northland Bank in Canada

the insurer of their deposits to make up the capital shortfall. Note that since the ratio of equity to loan capital is so low in banks, shareholders bear large risks of capital loss. See the cross-hatched area of Figure 1b which represents the payment to shareholders for risk-bearing. This implies that high average returns to equity in banks are reasonable. Large risks borne must be compensated for. The target value of twenty percent does not seem usurious, given the usual leverage observed.

8. An Equity Financed Lending Institution

Though all loan portfolio volatility would be borne by shareholders in a purely equity financed institution, the ratio of loan capital to equity capital would be unity, *ex ante*, lower than for the traditionally structured bank above. Risk per dollar of equity capital would be lower, given similar prudence in lending. Hence, *ceteris paribus*, the cost of capital would be lower than it is with the traditional form for a bank. Depositors would have a separate institution serving their check-writing and safe-keeping requirements and their deposit capital would be held by the institution in safe securities, earning the market rate, r . We sketch an equilibrium for a bank (now two institutions) in Figures 3a and 3b. These are to be compared with the analogues in Figures 1a and 1b.

was reflected in the value of equity before the failure in fact occurred. They also consider the collapse of the Bank of British Columbia, which hovered on collapse but was bought before bankruptcy occurred.

The cross-hatched area is compensation to shareholders for risk-bearing. Of course to achieve the same volume of lending under the two arrangements, much more equity capital would be needed when deposit reserves were implicitly set at one hundred percent. This would cause the price of equity to be bid up. For a comparable volume of lending, it is not obvious which institutional form would involve a lower cost of capital for lending. With the traditional bank form, γ is relatively high because of leverage whereas with the other form γ would be relatively high because of increased demand for equity capital.

An alternative view is that banks would end up doing less lending when financed purely by equity. This conforms with the cliché: deposit capital is a cheap source of loan capital for a bank. Savings would be allocated to investment projects through different channels. Whether the nature of investment and overall economic growth would be affected is of course the large, open, and interesting question. There would be no sharing of K-capital under separate institutions for loan-placing and deposit-taking. This seems like a fairly high cost of operating the one hundred percent equity financed form of bank. The advantage is, as Irving Fisher emphasized, that depositors would be free of risk of capital loss and no deposit insurance would be required.

9. Rate (price) Determination

We focused on accounting in the banking sector and on a price-taking representative banking firm. Industry equilibrium was neglected. We left open the formation of prices (i.e. r_l , r_d , r_{dd} , r , s , and i). At a minimum, we need a market demand schedule for bank loans and a market demand schedule for the transaction and safe-keeping services provided to depositors by the bank. The demand for bank

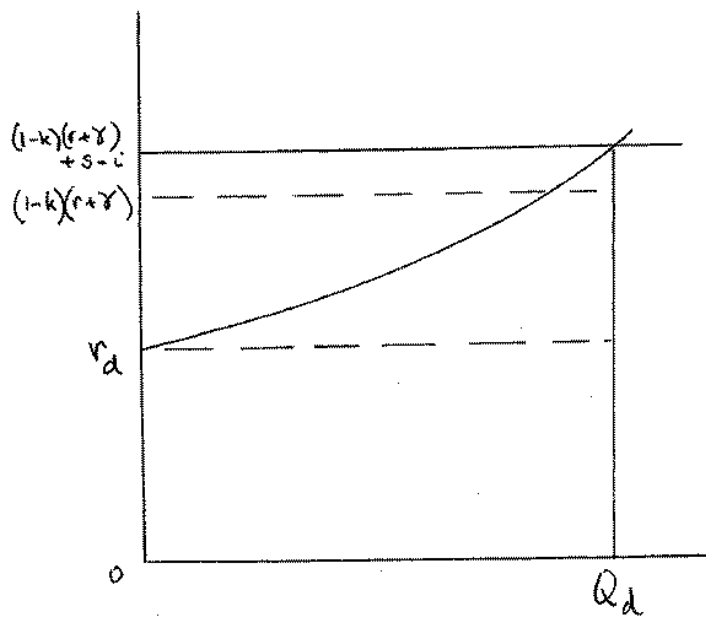
loans will depend on the relative cost of finance from alternative sources and on the current "need" for investment by firms and consumer credit by households. Similarly, the demand for transactions services from banks depends on the relative costs of transacting with cash alone and credit cards (i.e. without checks). Also the demand depends on the level of economic activity or the "need" for transactions services. The volume of transactions services provided by the banks increases with the inflation rate, at least for high rates of inflation.

The certain or market rate r is determined in the larger (economy-wide) market for credit; in the balancing of aggregate savings with investment demand. Bank rates are defined in submarkets. The economic analysis of aggregates generally glosses over the details of portfolio management by banks. It is an open question whether predictions from macroeconomic analysis suffer greatly from this approach. Tobin moved from the analysis of aggregates to the analysis of flows in the financial sector as he moved his research forward. He also developed a model of a commercial bank (Tobin [1982]). It is also interesting to observe that younger enthusiasts of "Keynesian economics" consider that business downturns are initiated by monetary phenomena (Krugman [1994; pp. 26-33]). The analysis of banking activity will presumably continue to flourish in this intellectual climate.

10. Concluding Remarks

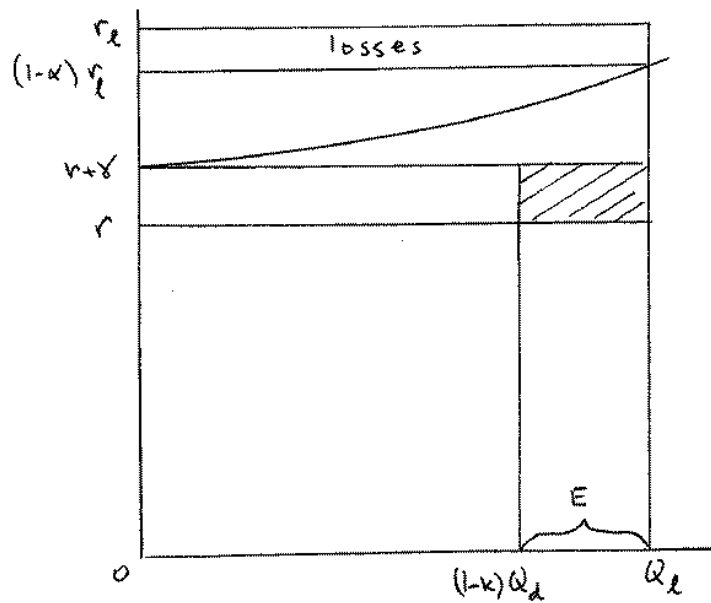
Once one recognizes that equity is required to close the bank's capital "budget constraint", certain basic accounting results follow. The value of a bank's activity decomposes naturally into a part for deposit-processing activity and a part for loan-placing activity. The role of equilibrium loan losses must be dealt with as well as the allocation for shareholder risk. We found that these could be dealt with

in a natural way also. This then represents a contribution to the analysis of risk in the national accounts. We were forced to construct a model of a representative bank in order to "explain" our accounting entries or to provide an economic basis for them. This led us into the analysis of the value of deposit capital to the bank and the allocation of loan risk to shareholders and depositors. We observed the novel phenomenon: though borrowers implicitly compensate depositors for risk-bearing in their payment for loans, depositors do not receive the payment; it ends up being used by the bank to expand deposit-taking, at the margin. In our reckoning, this provided an "explanation" for the view that deposits are a "cheap" source of funds for lending to the bank.



1a

Figure



1b

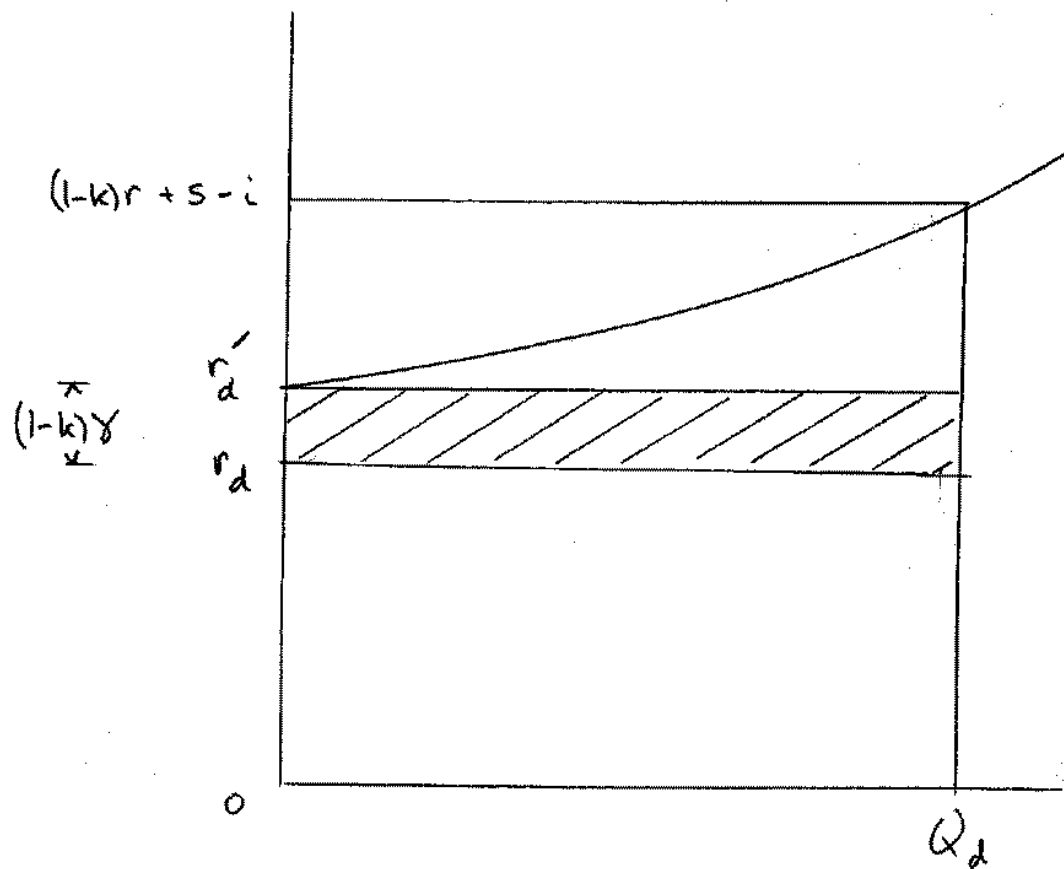
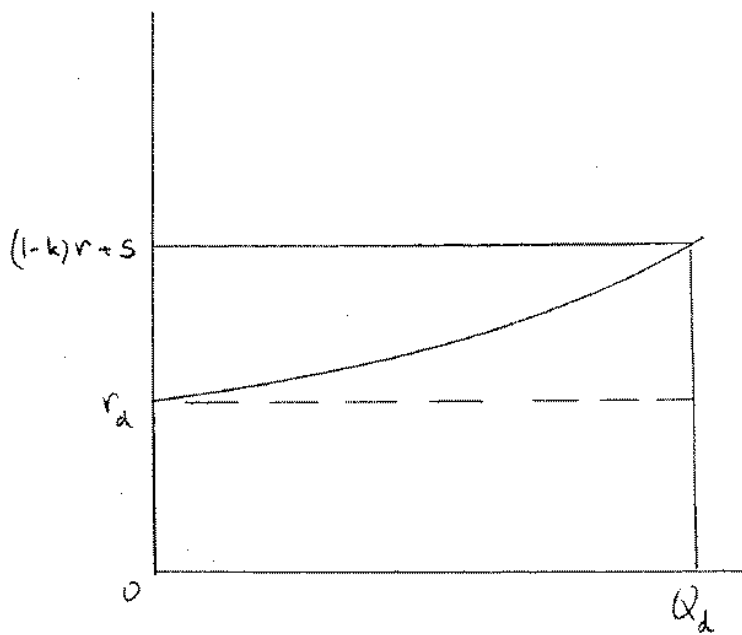
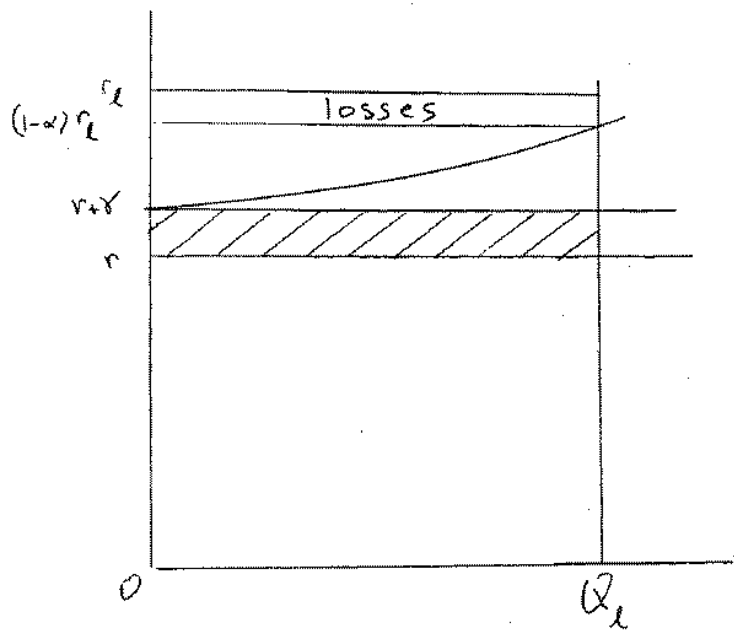


Figure 2



3a

Figure



3b

TABLE 1
Constructed Net Interest Flow

Year	Loans $[r_l - r] Q_l$	Term $[r - r_d] Q_d$	Non-checkable savings $[r - r_d] Q_d$	Checkable savings $[r - r_d] Q_d$	Demand $[r - r_d]$ times Q_{dd}	kQ_{dd}	rE	Net	Net* B of C	Δ
1981	4747.38	862.5	427.9	863.55	2893.9	237.3	1795.8	11355.93	7950	3405.9
1982	5561.15	902.0	469.9	663.87	2001.1	166.62	1700.7	11132.12	8456.2	2675.9
1983	5104.93	861.4	510.6	490.27	1495.6	121.66	1287.9	9628.7	9853.5	-224.8
1984	3841.48	904.7	499.6	897.60	1709.13	139.5	1817.78	9531.1	9997.7	-466.6
1985	4446.76	989.3	446.1	1442.69	1430.22	116.93	1735.77	10373.9	10520.6	-146.7
1986	5528.29	1044.0	423.5	1891.69	1411.30	116.56	1863.05	12045.3	11422.4	622.9
1987	5514.79	1078.2	482.9	1828.3	1481.46	121.38	1747.6	12011.8	12100.2	- 88.4
1988	6506.76	1200.2	575.8	2075.26	1896.03	153.22	2401.8	14500.7	13893.2	607.5

*Source: Bank of Canada Review, February, Chart K2, 1990, 1986.

TABLE 1a
DATA and SOURCES for TABLE 1

Year	r prime	r ₉₀	r _{ad}	E	Q _i	Q _{ad}	Checkable Savings	Non-checkable savings	Term
1981	19.29	17.77	2.6	10105.9	157198	19076.5	6226	42787.5	43124.5
1982	15.81	13.81	2.2	12315.4	158890	17235.8	6699.42	46992.8	45099.0
1983	11.17	9.32	1.3	13819.7	152386	18647.9	8011.50	51054.8	43069.6
1984	12.06	11.10	1.58	16376.4	156795	17953.3	11192.8	49997.0	45286.8
1985	10.58	9.46	1.36	18387.4	168438	17657.8	21858.9	44613.5	49464.5
1986	10.52	8.99	1.37	20793.0	180663	18521.8	30910.1	42352.8	52191.9
1987	9.52	8.17	1.19	21338.2	194869	21224.4	33362.4	48290.8	53908.6
1988	10.83	9.42	1.26	25496.4	223600	23235.7	31160.9	57584.8	60011.1
Source:	Cansim B113855	Cansim B113851	Cansim D88568 D88598	BC K2	Cansim B634	Cansim B685	Cansim B678	Cansim B679	Cansim B680

TABLE 1b
Interest Rate Gaps

Year	\bar{r}_d	$r - \bar{r}_d$	$r_l - r$	$r_l - \bar{r}_d$
1981	15.89	1.88	3.02	4.9
1982	12.34	1.47	3.50	4.97
1983	8.1	1.22	3.35	4.57
1984	9.67	1.43	2.46	3.89
1985	8.03	1.43	2.62	4.05
1986	7.52	1.47	3.03	4.50
1987	6.78	1.39	2.85	4.24
1988	7.97	1.45	2.91	4.36

REFERENCES

- Barnett, W.A. [1978], "The User Cost of Money" *Economics Letters* 1, pp. 145-49.
- Barnett, W.A. [1981], *Consumer Demand and Labor Supply: Goods, Monetary Assets and Time*, Amsterdam: North Holland.
- Berger, A. and T.H. Hannan [1989], "The Price-concentration Relationship in Banking", *Review of Economics and Statistics*, 71, May, pp. 291-99.
- Berger, Allen N. and David B. Humphrey [1992], "Measurement and Efficiency in Banking" in . Z. Griliches, ed. *Output Measurement in the Service Sectors*, Chicago, University of Chicago Press (National Bureau of Economic Research), pp. 259-79.
- Booth, L. [1996], "Competition and Profitability in the Financial Services Industry in Canada", (typescript).
- Crouhy, M. and D. Galas [1991], "A Contingent Claim Analysis of a Regulated Depository Institution", *Journal of Banking and Finance*, 15, pp. 73-90.
- Diewert, E. [1994], "Functional Form Problems in Modelling Insurance and Gambling", UBC Economics Discussion Paper #94-31.
- Fixler, D. and, K.D. Zieschange [1991], "Measuring the Nominal Value of Financial Services in the National Income Accounts" *Economic Inquiry*, 29, January, pp. 53-68.
- Giammarino, R., E. Schwartz, and J. Zechner [1989], "Market Valuation of Bank Assets and Deposit Insurance in Canada", *Canadian Journal of Economics*, 22, 1, February, pp. 109-127.
- Hancock, D. [1985], "The Financial Firm: Production with Monetary and Nonmonetary Goods", *Journal of Political Economy*, 93, 5, pp. 859-880.
- Hannan, T.H. [1991], "Bank Commercial Loan Markets and the Role of Market Structure: Evidence from Surveys of Commercial Lending", *Journal of Banking and Finance*, 15, February, pp. 133-149.
- Hartwick, J.M. [1996], "The Value of Limited Liability in Shareholder Equity", (typescript).
- Krugman, Paul [1994], *Peddling Prosperity*, New York: Norton.
- Miller, M.H. [1995], "Do the M & M Propositions Apply to Banks?", *Journal of Banking and Finance*, 19, pp. 483-89.

- Rose, Peter [1994], *Money and Capital Markets: The Financial System in an Increasingly Global Economy*, Burr Ridge, Ill.: Irwin.
- Schmjer, J. [1992], *Banks Under Stress*, Paris: Organization for Economic Cooperation and Development (OECD).
- Tobin, J. [1982], "The Competitive Banking Firm, A Simple Model", *Scandinavian Journal of Economics*, 84, 4, pp. 495 - 530.
- Triplett, Jack E. [1992], "Comment" in Z. Griliches, ed. *Output Measurement and the Service Sectors*, Chicago: University of Chicago Press (National Bureau of Economics Research) pp. 287-300.